Approximately linear phase notch filters with magnitude characteristic symmetry

Goran Stančić, Ivana Kostić, Miloš Živković and Ivan Krstić

Abstract - A new method for design and software realization of approximately linear phase notch filters with magnitude characteristic symmetry is presented in this paper. The proposed configuration has parallel nature with pure delay in one path and allpass filter in the another. The resulting filter fulfills all predefined specifications in a wide range of maximal allowed attenuation i.e. for arbitrary given maximal passband attenuation and stopband width, passband boundary frequencies are symmetrical about given notch frequency. The efficiency of the presented method is illustrated on few examples.

Keywords - Notch filter, allpass filter, parallel connection, comb filter.

I. Introduction

The notch filters are invariably used in communication, control, instrumentation, and bio-medical engineering, besides a host of other fields, to eliminate noise and power line interferences [1-5]. The notch filter highly attenuates a particular frequency component in the input signal while leaving nearby frequency components in the ideal case unchanged. For example, the elimination of a sinusoidal interference corrupting a signal, such as the 50 Hz power-line interference in the design of a digital instrumentation system, is typically achieved with a notch filter tuned to the frequency of the interference. Usually very narrow notch characteristic is desired to filter out the single frequency or sinusoidal interference without distorting the signal of interest [6]. The notch filters find implementation in both, analog and digital domain. Analog implementations consume less bandwidth, but the signals are more likely to get deteriorated during transmission. In contrast, digital filters are generally noise-immune and flexible in implementation, although consume a lot more bandwidth to carry the same information [1][7].

Digital notch filters can be designed as infinite impulse response (IIR) as well as finite impulse response (FIR) structures. In situations where linearity of the phase is not important, IIR filters are preferred since these require much lower order than corresponding the FIR ones for the same set of magnitude response specifications. The standard analog notch filter can be starting point to obtain corresponding digital IIR notch filter by using bilinear transformation for example. One of the major problem in IIR filters design is nonlinear phase response and, therefore, phase distortion introduced in general [8]. However, it is possible to reduce phase distortion by cascading an all-pass phase equalizer [6].

FIR filters, on the other hand, are unconditionally stable and can be designed to provide exact linear phase characteristics. They find extensive use in applications where frequency dispersion due to nonlinear phase is undesirable, such as in speech processing, digital communication, image processing, etc.. This is the specific reason that a large number of commercial chips carry out signal processing with FIR filters. Standard FIR filter design methods, such as windowing, frequency sampling and computer-aided/optimization may be used for designing FIR notch filters. However, most of these methods result in ripples in the passbands [9].

The rest of the paper is structured as follows. In the Section II, the basic relations are derived between notch magnitude and allpass filter phase. A method for determination of filter coefficients and few examples are done in Section III. The results of the simulation of designed filter performance are given in Sections IV and V along with difference equations implemented in the Matlab® in order to obtain output of the notch filter.

II. Problem definition

In this section the synthesis of linear phase filters with arbitrary number of notch frequencies with magnitude characteristic symmetry will be discussed. The notch filter will be realized as parallel connections of two allpass sub-filters. The coupled allpass structure offers convenient way to solve the problem of filter design thanks to straightforward dependence of notch filter's magnitude and phase characteristic of corresponding allpass sub-filter. A problem of design of notch filter magnitude is easy to reformulate as the allpass filter phase approximation problem.

The realized notch frequency will be positioned exactly at predefined location, while at the same time cut-off edge frequencies are symmetrical regardless the value of the notch frequency. Practically, the full control of the magnitude characteristic at three points is achieved in vicinity of every notch frequency [2]. The location of notch

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frequency ω_n, maximal allowed attenuation in passbands a (given in dB) and stopband width B_w are design input parameters. Stopband lower ω_l and upper ω_r edge frequencies are

$$\omega_{l_i} = \omega_{n_i} - B_{w_i}/2$$
$$\omega_{r_i} = \omega_{n_i} + B_{w_i}/2 \quad (1)$$

based on the mentioned symmetry. Usually, for attenuation at cut-off frequencies the value of 3 dB is adopted. The proposed method allows arbitrary positive value for maximal attenuation in passbands to be chosen.

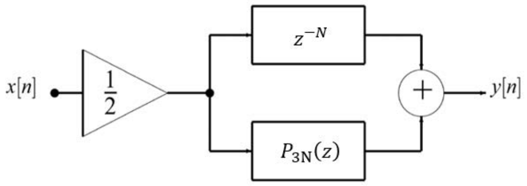

Fig. 1. Realization structure for linear phase filters with N notch frequencies using parallel connection of two alpass sub-filters

The transfer function of the proposed linear phase notch filter, realized using pure delay and allpass sub-filter, presented in the Fig. 1, can be written in the next form

$$H(z) = \frac{1}{2}[z^{-N} + P_{3N}(z)] \quad (2)$$

This solution also allows realization of complementary filter for signal extraction, using only one additional adder [10]. Transfer function of complementary filter is

$$G(z) = \frac{1}{2}[z^{-N} - P_{3N}(z)] \quad (3)$$

and it is represented in the Fig. 2.

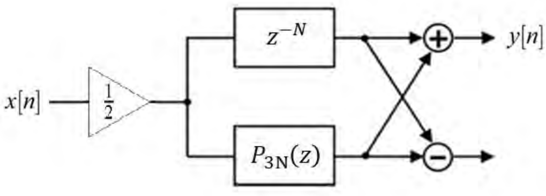

Fig. 2. Realization of complementary filter

The transfer function of subfilter $P_{3N}(z)$ is given with

$$P_{3N}(z) = \frac{p_{3N} + \cdots + p_1 z^{-(3N-1)} + z^{-3N}}{1 + p_1 z^{-1} + \cdots + p_{3N} z^{-3N}}$$

$$P_{3N}(z) = z^{-3N}\frac{D(z^{-1})}{D(z)} \quad (4)$$

$$D(z) = 1 + p_1 z^{-1} + \cdots + p_{3N} z^{-3N}$$

The phase of allpass filter $P_{3N}(z)$ is

$$\varphi(\omega) = -3N\omega + 2atan\frac{p_1\sin(\omega) + \cdots + p_{3N}\sin(3N\omega)}{1 + \cdots + p_{3N}\cos(3N\omega)} \quad (5)$$

By substituting Eq. (4), alongside with $z = \exp(j\omega)$, into Eq. (2), after simple mathematical manipulations the magnitude characteristic of the notch filter could be obtained

$$|H(e^{j\omega})| = \left|\cos\frac{-N\omega - \varphi(\omega)}{2}\right| \quad (6)$$

The phase of notch filter has value

$$argH(e^{j\omega}) = \frac{-N\omega + \varphi(\omega)}{2} \quad (7)$$

Proposed filter's phase approximates piecewise linear phase in all passbands. In order to provide a symmetric magnitude characteristic of notch filter it is necessary that the number of coefficients we determine to be three times greater than given number of notch frequencies. This approach allows that the maximum attenuation in all passbands can be independently controlled. Maximum phase approximation error e_i in the i-th passband is given by Eq. (8) where a_i is attenuation in i-th passband given in dB.

$$e_i = 2arccos(10^{-a_i/20}) \quad (8)$$

III. DETERMINATION OF FILTER COEFFICIENTS

Structure for realization of filter with one notch frequency is composed of first order delay and third order allpass phase corrector which is explained in detail in paper. Described approach can be generalized and applied to filter with N notch frequencies. In Fig. 3 points of interest are marked, notch frequency ω_n and passband egde frequencies ω_l and ω_r. Based on these frequencies, a system of equations is formed to determine the coefficients of filter [11].

The filter with one notch frequency, which structure is given in Fig. 1, can be made in two ways. The first approach is that bare wire i.e. zero-order delay line be in one path and second order allpass section in another path of parallel structure. Only two coefficients need to be determined i.e. to form two equations. It is valid to use two points of interest ω_n and ω_r for $\omega_n < 0.5\pi$ or ω_n and ω_l for $\omega_n > 0.5\pi$. In that case, magnitude characteristic will not be symmetric. Allpass filter with no multiple poles does not offer an equiripple phase approximation. In the other case, if an eqqiripple approximation of the linear phase should be achieved, allpass filter must be at least of 4th order. Double pair of complex conjugate poles will ensure the phase jump of 2π at notch frequency.

An explanation for determining the coefficients of filter with two notch frequencies is given below. The structure of the filter is consisted of 2nd order pure delay and 6th order allpass filter. The phase of proposed notch filter is a monotonically decreasing function of frequency. Phase approximation error is also monotonic function with maximum approximation error located at boundary frequencies, as given in Fig. 4.

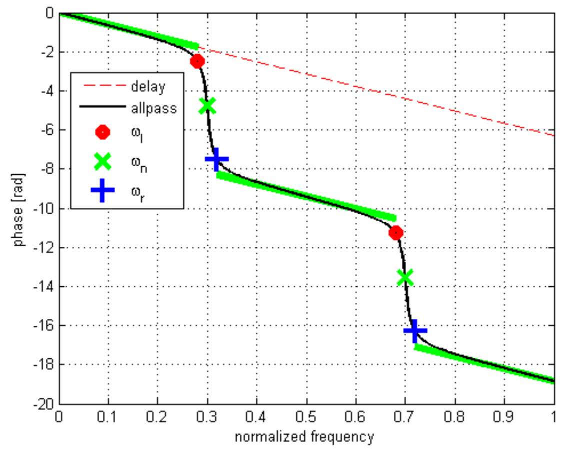

Fig. 3. Phase of delay line, allpass filter and points of interest ω_n, ω_l and ω_r for filter with two notch frequencies

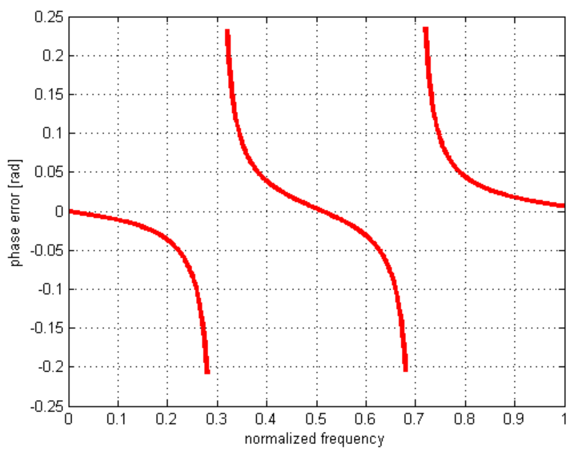

Fig. 4. Phase error of notch filter with two notch frequencies

According to Fig. 4, for IIR filter with two notch frequencies, next set of equations could be defined.

$$-2\omega_{n_1} - \pi = -6\omega_{n_1} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{n_1})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{n_1})} \qquad (9)$$

$$-2\omega_{l_1} - e = -6\omega_{l_1} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{l_1})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{l_1})} \qquad (10)$$

$$-2\omega_{r_1} - 2\pi + e = -6\omega_{r_1} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{r_1})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{r_1})} \qquad (11)$$

$$-2\omega_{n_2} - 3\pi = -6\omega_{n_2} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{n_2})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{n_2})} \qquad (12)$$

$$-2\omega_{l_2} - 2\pi - e = -6\omega_{l_2} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{l_2})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{l_2})} \qquad (13)$$

$$-2\omega_{r_2} - 4\pi + e = -6\omega_{r_2} + 2arctg\frac{\sum_{i=1}^{6} p_i \sin(i\omega_{r_2})}{1+\sum_{i=1}^{6} p_i \cos(i\omega_{r_2})} \qquad (14)$$

Which could be given in matric form

$$\boldsymbol{Ap} = \boldsymbol{b} \qquad (15)$$

from which coefficients

$$\boldsymbol{p} = [p_1 \quad p_2 \quad \cdots \quad p_{3N}]^T \qquad (16)$$

will be obtainted.

The proposed approach could be generalized and applied to case of IIR notch filter with arbitrary number of notch frequencies N. After some manipulations, matrix $\boldsymbol{A}$ of sistem of Eq. (9) - Eq. (14) in case of N notch frequencies can be given with

$$\begin{aligned} A(i,j) &= (-1)^{i+1}\cos\left((2N-j+1)\omega_{n_i}\right) \\ A(N+i,j) &= (-1)^{i+1}\sin\left((2N-j+1)\omega_{l_i} + e/2\right) \\ A(2N+i,j) &= (-1)^{i}\sin\left((2N-j+1)\omega_{r_i} - e/2\right) \\ i &= 1,2,\dots,N \qquad j = 1,2,\dots,3N \end{aligned} \qquad (17)$$

Elements of vector $\boldsymbol{B}$ have value

$$\begin{aligned} b(i) &= (-1)^{i}\cos\left(N\omega_{n_i}\right) \\ b(N+i) &= (-1)^{i+1}\sin\left(N\omega_{l_i} - e/2\right) \\ b(2N+i) &= (-1)^{i}\sin\left(N\omega_{r_i} + e/2\right) \\ i &= 1,2,\dots,N \end{aligned} \qquad (18)$$

IV. Examples

For different values of stopband width B_w and allowed attenuation a in passband below are presented two figures which give position of the poles and zeros of notch filter with four notch frequencies $\omega_{n_i} = 0.1\pi$, 0.2π, 0.4π and 0.8π.

According to the obtained results, displayed in Fig. 5 and 6, one can conclude that the reducing allowed attenuation in the passband make poles and zeros to approach closer to the unit circle. It can cause a problem in some implementations with restricted number of bits dedicated for filter coefficients representation because poles are too close to the edge of stability region and can leave the unit circle.

In case $B_w = 0.09\pi$ and $a = 3$dB, phase characteristic of allpass sub-fitler is given in Fig. 7. It is noticeable that the highest phase slope is observed at the frequencies where the poles are located, i.e. around notch frequencies.

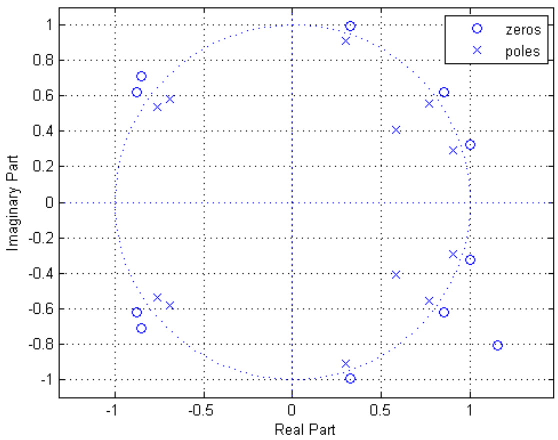

Fig. 5. Poles and zeros of allpass filter, $B_w = 0.09\pi$ and $a = 3$ dB

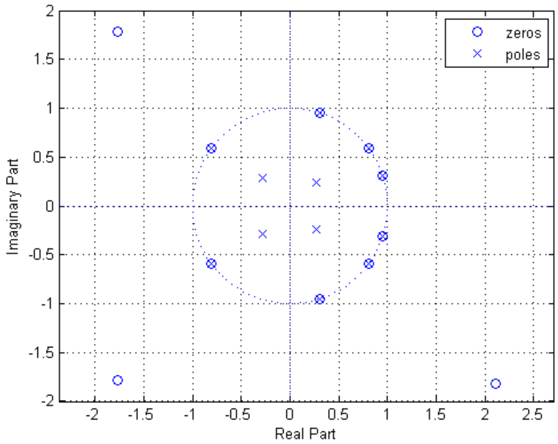

Fig. 6. Poles and zeros of allpass filter, $B_w = 0.09\pi$ and $a = 0.01$ dB

For any set of notch filter input specifications system of equations Eq. (15) can be solved. As mentioned earlier, one can not control passband magnitude. For some input parameters (when one notch frequency is far away from another) maximal obtained attenuation in passband could surpass the given value at passband boubdary frequency.

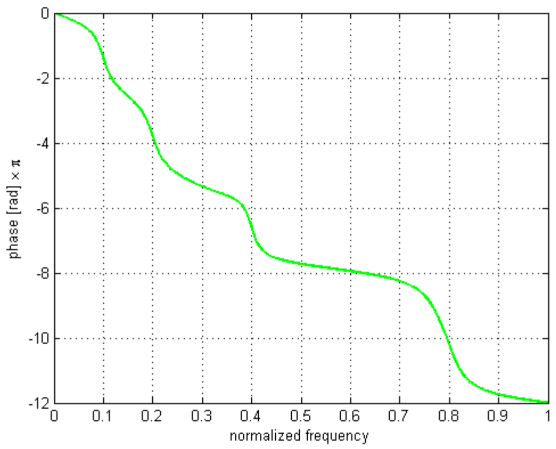

Fig. 7. Typical phase characteristic of allpass sub-filter

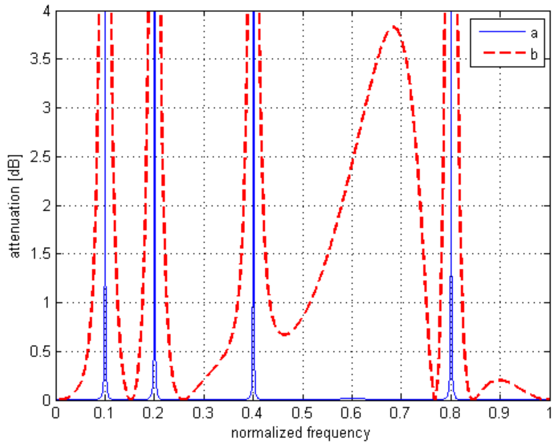

Fig. 8. Attenuation of filter with four notch frequencies, $B_w = 0.09\pi$ a) $a = 0.01$ dB and b) $a = 3$ dB

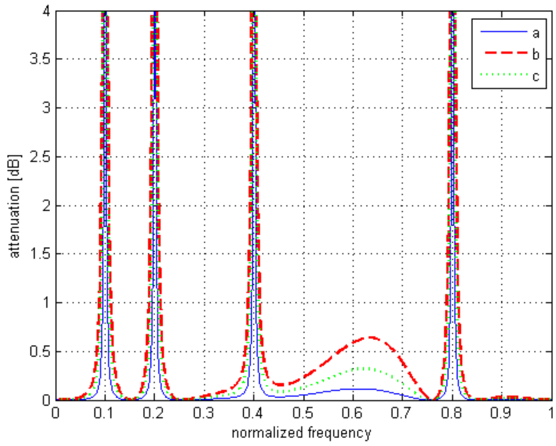

Fig. 9. Influence of bandwidth B_w on attenuation curve for $a = 1$ dB a) $B_w = 0.09\pi$, b) $B_w = 0.07\pi$ and c) $B_w = 0.03\pi$

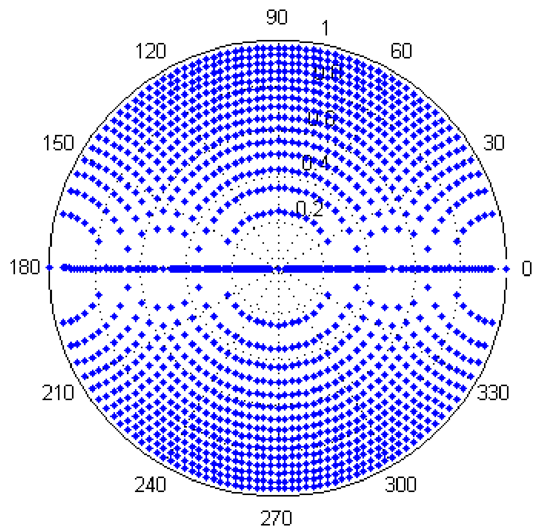

Fig. 10. Possible position of allpass filter poles for four bit fixed-point fraction representation

The Table I and Fig. 9 show that by reducing the value of width of stopband B_w and attenuation a at passband edges, the maximum attenuation in the passband (which is out of control by system of equations) is also reduced, but that the ratio a/a_{max} changes very little in the wide range of parameters B_w and a.

TABLE I

MAXIMAL ATTENUATION IN PASSBAND a_{max} AND a/a_{max} FOR DIFFERENT VALUES OF BANDWIDTH B_w AND ATTENUATION a

a_{max} \ a/a_{max}		a [dB] 3	1	0.1	0.01
B_w	0.09π	3.83 / 0.78	1.11 / 0.90	0.12 / 0.85	0.01 / 0.81
	0.07π	2.17 / 0.78	0.64 / 1.56	0.01 / 1.51	0.01 / 1.46
	0.05π	1.10 / 2.73	0.32 / 3.13	0.03 / 3.13	0.003 / 3.04
	0.03π	0.42 / 7.23	0.12 / 8.55	0.02 / 8.55	0.01 / 8.77

It can be seen from Fig. 10 that significant errors are possible in the implementation of the filter with the poles with low phase angles. This problem can be solved by larger number of bits dedicated for coefficients representation as well as spesific filter structures which go beyond the scope of this paper.

Described approach provides a solution for a wide range of input parameters (number of notch frequencies N, stopband width B_w and allowed attenuation a) which is shown in Table I. It can be seen from the table (for $B_W = 0.09\pi$) that if the bandwidth is too large, the maximal obtained attenuation in the passband (3.83 dB, which one can not control) may be greater than the prescribed attenuation (given at the boundaries of the passband (3 dB) and which need to be extremal value).

The proposed procedure can be also successfully used for realization of comb filters, as shown in the Fig. 11 for the case of a filter with nine notch frequencies $\omega_{n_i} = i\pi/10$.

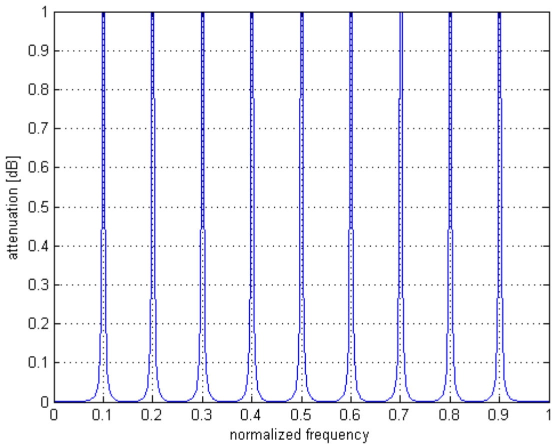

Fig. 11. Attenuation of the comb filter with nine notch frequencies $\omega_{n_i} = i\pi/10$, a =0.5 dB, $B_w = 0.04\pi$

V. SOFTWARE REALIZATION AND THE SIMULATION RESULTS

Power systems are designed to operate at frequencies of 60 (America and part of Asia) or 50Hz (in large parts of the world). However, certain types of loads (nonlinear loads) produce currents and voltages with frequencies that are integer multiples of the 50 or 60 Hz fundamental frequency. These higher frequencies are a form of electrical pollution known as power system harmonics. Notch filters have the application to remove these unwanted components.

The Matlab® software package is used for design and realization of the proposed linear phase notch filters with magnitude characteristic symmetry. Sinusoidal noise with amplitude 0.2, at power-line frequency F_n = 50 Hz, 100 Hz and 150 Hz are superimposed on the electrocardiogram (ECG) signal $s[n]$ downloaded from the database MIT-BIH [12], as it is given in Eq. (19). All available signals in MIT-BIH database are recorded after digitalization using sampling frequency F_s = 360 Hz.

Specifications of the digital filter in z domain are: $\omega_n = 2\pi\ F_n\ /\ F_s = 5\pi/18$, while for stopband width and maximal attenuation in passbands are adopted $B_w = 0.02\pi$ and a = 1 dB, respectively.

The signal duration is 10 seconds so n = 0, 1, … , 3599.

$$x[n] = s[n] + \sum_{i=1}^{3} 0.2\sin(2\pi i 50 n/F_s) \tag{19}$$

$$y[n] = \frac{1}{2}[x[n-3] + \omega[n]] \tag{20}$$

$$\omega[n] = x[n-9] + \sum_{i=1}^{9} p_i\,(x[n-9+i] - \omega[n-1]) \tag{21}$$

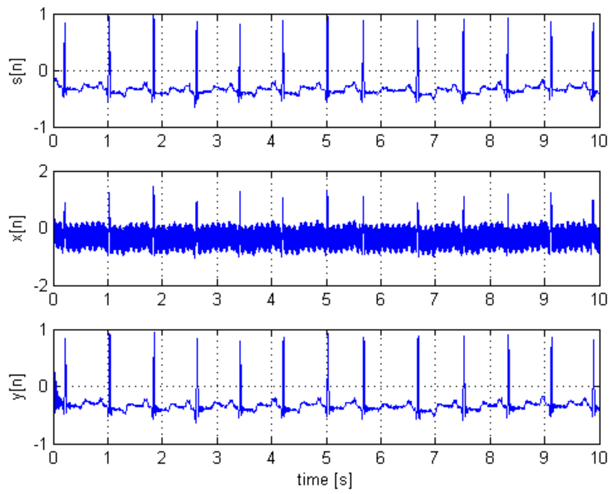

Fig. 12. ECG signal $s[n]$, ECG signal with added sinusoidal noise $x[n]$ and filtered signal at output of notch filter $y[n]$

In Fig. 12, ECG signal $s[n]$ which corresponds to 100^{th} sample in MIT-BIH database is shown, together with corrupted version of this signal $x[n]$ after addition of sinusoidal noise. Filtered version of the signal at output of proposed notch filter $y[n]$ is also presented in Fig. 12.

It is evident that designed notch filter successfully eliminated sinusoidal noise at power-line frequency from ECG signal. In the steady state, output signal $y[n]$ is free of noise. A deviation at the beginning of output signal can be observed. It is normal behavior of digital filters since digital filter output data are valid after filter latency.

VI. Conclusion

Design and software realization of IIR digital filter with arbitrary number of notch frequencies (N) is presented in this paper. The proposed filter is realized as parallel structure with Nth order pure delay in one path and allpass sub-filter of order 3N in another path. Parallel connection enables realization of a complementary filter with only one additional adder and this solution exhibits low sensitivity on coefficients quantization.

Unlike existing methods for design of notch filter, the proposed configuration delivers solution with cut-off frequencies symmetry about notch frequency i.e. for arbitrary given maximal passband attenuation and stopband width, passband boundary frequencies are symmetrical about given notch frequency. The resulting filter has approximately linear phase in passbands and minimal order to provide symetry.

The functionality of the presented model is illustrated by filtering ECG input signal corrupted with sinusoidal noise at the power-line frequency and first two harmonics. The proposed method could be also applied on design of comb filter or filter for extraction of signal components at frequencies of interest.

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